Licenciaturas em Engenharia de Energias Renováveis

Matemática I - 2011/2012

2º teste Parcial - 17 de Janeiro 2012

Docente: Carlos Balsa - Departamento de Matemática - ESTiG

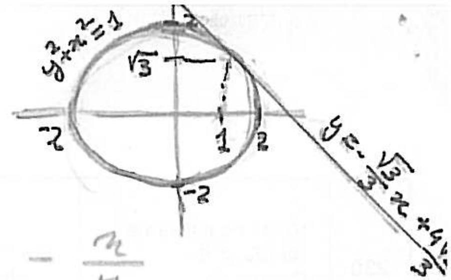
Instruções:

- Apenas é permitido consultar o formulário.
- Não é permitida a utilização de máquina de calcular.
- Explicite detalhadamente todos os cálculos efectuados.

Duração: 1h45

1. Determine a equação da recta tangente à circunferência de equação $y^2 + x^2 = 4$ no ponto $(1;\ \sqrt{3})$.
2. Derive a seguintes funções:
   (a) $f(x) = e^{2x}\cos(x)$
   (b) $f(x) = \ln(x\sqrt{4+x})$
   (c) $f(x) = \frac{5x+3}{4x^2-7}$
3. Calcule os seguintes integrais definidos
   (a) $\int_0^\pi x\cos(x)\,dx$
   (b) $\int_1^2 \frac{1}{2x^2} - 5x^3 + \sqrt{x}\,dx$
   (c) $\int_0^1 \frac{2x+3}{(x^2+3x-7)^2}\,dx$
   (d) $\int_{-1}^0 \frac{1}{4-5x}\,dx$
   (e) $\int_1^3 e^{-4x}\,dx$
4. Considere as funções $f(x) = 4 - x^2$ e $g(x) = x + 2$
   (a) Faça os gráficos destas duas funções, indicando os pontos de intersecção
   (b) Calcule a área compreendida entre os dois gráficos
   (c) Calcule o volume definido pela rotação do gráfico de $f(x)$, desde $x = -2$ até $x = 2$, em torno do eixo x

1



$$(y^2+x^2)' = (4)' \Leftrightarrow 2y \cdot y' + 2x = 0$$

$$\Leftrightarrow y' = -\frac{2x}{2y} \Leftrightarrow y' = -\frac{x}{y}$$

$$y^2 + 1 = 4 \Leftrightarrow y^2 = 3 \Leftrightarrow y = \sqrt{3}$$

$$y'(1, \sqrt{3}) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$m = \frac{y - y_0}{x - x_0} \Leftrightarrow m(x - x_0) = y - y_0 \Leftrightarrow y = m(x - x_0) + y_0$$

$$y = -\frac{\sqrt{3}}{3}(x - 1) + \sqrt{3} \Leftrightarrow y = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} + \sqrt{3}$$

$$\Leftrightarrow \boxed{y = -\frac{\sqrt{3}}{3}x + \frac{4\sqrt{3}}{3}}$$

Verificação: Se $x = 1 \Rightarrow y = -\frac{\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} = \frac{3\sqrt{3}}{3} = \sqrt{3}$ ✓

2

a) $f(x) = e^{2x}\cos(x)$

$$f'(x) = (e^{2x})'\cos(x) + e^{2x}(\cos(x))' = 2e^{2x}\cos(x) + e^{2x}(-\operatorname{sen}(x))$$

$$= e^{2x}(2\cos(x) - \operatorname{sen}(x))$$

b) $f(x) = \ln(x\sqrt{4+x})$

$$f'(x) = \frac{(x\sqrt{4+x})'}{x\sqrt{4+x}} = \frac{\sqrt{4+x} + x\frac{1}{2\sqrt{4+x}}}{x\sqrt{4+x}}$$

$$= \frac{\frac{2(4+x) + x}{2\sqrt{4+x}}}{x\sqrt{4+x}} = \frac{8+3x}{2x(4+x)}$$

c) $f(x) = \frac{5x+3}{4x^2-7}$

$$f'(x) = \frac{(5x+3)'(4x^2-7) - (5x+3)(4x^2-7)'}{(4x^2-7)^2} = \frac{5(4x^2-7) - (5x+3)(8x)}{(4x^2-7)^2}$$

$$= \frac{20x^2-35-(40x^2+24x)}{(4x^2-7)^2} = \frac{20x^2-35-40x^2-24x}{(4x^2-7)^2}$$

$$= \frac{-20x^2-24x-35}{(4x^2-7)^2}$$

3 a)

$$\int x\cos(x)\,dx = \operatorname{sen}(x)\,x - \int \operatorname{sen}(x)\,dx$$

$$= \operatorname{Sen}(x)\,x - \left(-\cos(x)\right)$$

$$= \operatorname{Sen}(x)\,x + \cos(x)$$

Verificação:

$$\left(\operatorname{Sen}(x)\,x + \cos(x)\right)' = \cos(x)\,x + \operatorname{Sen}(x) - \operatorname{Sen}(x) = \cos(x)(x)$$

$$= x\cos(x)$$

$$\int_0^{\pi} x\cos(x)\,dx = \left[x\operatorname{Sen}(x) + \cos(x)\right]_0^{\pi} = \pi\operatorname{sen}(\pi) + \cos(\pi) - 0 - \cos(0)$$

$$= \pi * 0 + (-1) - 1 = -2$$

— 4 —

b) $$\int \frac{1}{2x^2} - 5x^3 + \sqrt{x}\,dx$$

$$\frac{1}{2}\int x^{-2}\,dx - 5\int x^3\,dx + \int x^{\frac{1}{2}} = \frac{1}{2}\frac{x^{-1}}{(-1)} - 5\frac{x^4}{4} + \frac{x^{3/2}}{3/2}$$

$$= -\frac{1}{2x} - \frac{5}{4}x^4 + \frac{2}{3}\sqrt{x}^3$$

$$\int_1^2 \frac{1}{2x^2} - 5x^3 + \sqrt{x}\,dx = \left[\frac{-1}{2x} - \frac{5}{4}x^4 + \frac{2}{3}\sqrt{x}^3\right]_1^2 =$$

$$= \left(-\frac{1}{2} - \frac{5}{4}(2)^4 + \frac{2}{3}\sqrt{2}^3\right) - \left(\frac{1}{2} - \frac{5}{4}(1)^4 + \frac{2}{3}\sqrt{1}^3\right)$$

$$= \left(-\frac{1}{2} - \frac{5}{4}\times 16 + \frac{2}{3}\sqrt{8}\right) - \frac{1}{2} + \frac{5}{4} - \frac{2}{3} = -1 - 20 + \frac{4}{3}\sqrt{2} + \frac{5}{4} - \frac{2}{3}$$

$$= -21 + \frac{4}{3}\sqrt{2} + \frac{15-8}{12} = -21 + \frac{4\sqrt{2}}{3} + \frac{7}{12}$$

$$= \frac{-252 + 16\sqrt{2} + 7}{12} = \frac{16\sqrt{2} - 245}{12}$$

c) $$\int_0^1 \frac{2x+3}{(x^2+3x-7)^2}\,dx$$

$$\boxed{\begin{aligned} u &= x^2+3x-7 \\ du &= (2x+3)\,dx \\ u(0) &= -7 \\ u(1) &= 1^2+3(1)-7 = -3 \end{aligned}}$$

$$= \int_{-7}^{-3} \frac{1}{u^2}\,du = \int_{-7}^{-3} u^{-2}\,du$$

$$= \left[\frac{u^{-1}}{-1}\right]_{-7}^{-3} = \left[-\frac{1}{u}\right]_{-7}^{-3} = \left(-\frac{1}{-3}\right) - \left(-\frac{1}{-7}\right) = \frac{1}{3} - \frac{1}{7} =$$

$$= \frac{7-3}{21} = \frac{4}{21}$$

d)
$$\int_{-1}^{0} \frac{1}{4-5x}\,dx = -\frac{1}{5}\int_{-1}^{0} \frac{-5}{4-5x}\,dx = -\frac{1}{5}\Big[\ln|4-5x|\Big]_{-1}^{0}$$

$$= -\frac{1}{5}\Big[\ln|4| - \ln|4+5|\Big] = -\frac{1}{5}\big(\ln(4) - \ln(9)\big)$$

$$= -\frac{1}{5}\ln(2^2) + \frac{1}{5}\ln(3^2) = \frac{3}{5}\ln(3) - \frac{2}{5}\ln(2)$$

e) $$\int_1^3 e^{-4x}\,dx = -\frac{1}{4}\int_1^3 e^{-4x}(-4)\,dx$$

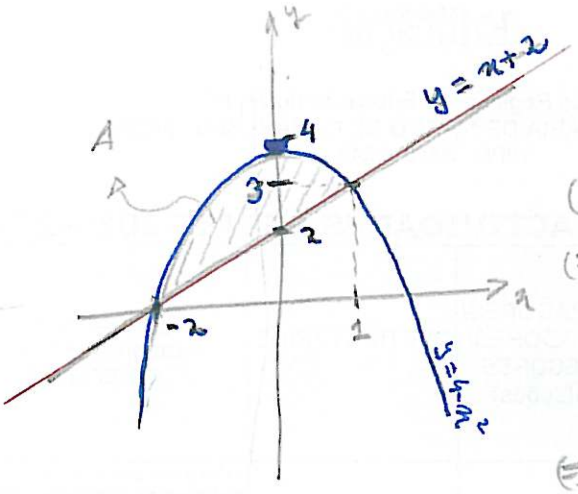
$$\boxed{\begin{aligned} u &= -4x \\ du &= -4\,dx \\ u(3) &= -12 \\ u(1) &= -4 \end{aligned}}$$

$$= -\frac{1}{4}\int_{-4}^{-12} e^{u}\,du = -\frac{1}{4}\Big[e^{u}\Big]_{-4}^{-12} = -\frac{1}{4}\left(e^{-12} - e^{-4}\right)$$

$$= \frac{e^{-4-8} - e^{-4}}{-4} = \frac{e^{-4}e^{-9} - e^{-4}}{-4} = \frac{e^{-4}(e^{-9}-1)}{-4} = \frac{e^{-4}(1-e^{-9})}{4}$$

4.

a) $f(x) = 4 - x^2$ e $g(x) = x + 2$



$4 - x^2 = x + 2$

$\Leftrightarrow -x^2 - x + 2 = 0$

$\Leftrightarrow x^2 + x - 2 = 0$

$x = \dfrac{-(1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2}$

$\Leftrightarrow x = \dfrac{-1 \pm \sqrt{1+8}}{2} = \dfrac{-1 \pm \sqrt{9}}{2}$

$\Leftrightarrow x = \dfrac{-1-3}{2} \vee x = \dfrac{-1+3}{2}$

$\Leftrightarrow x = -2 \vee x = 1$

b) $\text{Área} = A = \displaystyle\int_{-2}^{1} (4 - x^2) - (x+2)\, dx = \int_{-2}^{1} 4 - x^2 - x - 2\, dx = \int_{-2}^{1} -x^2 - x + 2\, dx$

$= \left[-\dfrac{x^3}{3} - \dfrac{x^2}{2} + 2x\right]_{-2}^{1} = \left(-\dfrac{1}{3} - \dfrac{1}{2} + 2\right) - \left(-\dfrac{(-8)}{3} - \dfrac{4}{2} - 4\right)$

$= -\dfrac{1}{3} - \dfrac{1}{2} + 2 - \dfrac{8}{3} + \dfrac{4}{2} + 4 = -\dfrac{9}{3} + \dfrac{3}{2} + 6 = 3 + \dfrac{3}{2}$

$= \dfrac{6+3}{2} = 9/2$

c)

$$V = \int_{-2}^{2} \pi (4-x^2)^2 \, dx = \pi \int_{-2}^{2} (16 - 8x^2 + x^4) \, dx$$

$$= \pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_{-2}^{2} = \pi \left(16 \times 2 - \frac{8 \times 8}{3} + \frac{32}{5} \right) - \pi \left(-16 \times 2 + \frac{8 \times 8}{3} + \frac{32}{5} \right)$$

$$= \pi \left(32 - \frac{64}{3} + \frac{32}{5} \right) - \pi \left(-32 + \frac{64}{3} + \frac{32}{5} \right)$$

$$= \pi \left(32 - \frac{64}{3} + \frac{32}{5} + 32 - \frac{64}{3} - \frac{32}{5} \right)$$

$$= \pi \left(64 - 2 \times \frac{64}{3} \right) = \pi \left(\frac{192 - 128}{3} \right)$$

$$= \pi \left(\frac{64}{3} \right) = \frac{64}{3} \pi$$