Matemática I, 2º teste parcial 2010/2011

1) $5x^3 - 2x^2\sqrt{y} + 4y^3 - 7 = 0$

a) $(5x^3)' - (2x^2\sqrt{y})' + (4y^3)' - (7)' = (0)'$

$\Leftrightarrow 15x^2 - \left(4x\sqrt{y} + 2x^2 \frac{y'}{2\sqrt{y}}\right) + 12y^2y' = 0$

$\Leftrightarrow 15x^2 - 4x\sqrt{y} - \frac{x^2}{\sqrt{y}} y' + 12y^2y' = 0$

$\Leftrightarrow y'\left(-\frac{x^2}{\sqrt{y}} + 12y^2\right) = 4x\sqrt{y} - 15x^2$

$\Leftrightarrow y' = \dfrac{4x\sqrt{y} - 15x^2}{12y^2 - \frac{x^2}{\sqrt{y}}}$

b) Se $x = 1$ e $y = 1 \Rightarrow y' = \dfrac{4(1)\sqrt{1} - 15(1)^2}{12(1)^2 - \frac{(1)^2}{\sqrt{1}}}$

$\Leftrightarrow y' = \dfrac{4 - 15}{12 - 1} = \dfrac{-11}{11} = -1$

$m = y'(1,1) = -1$

Eq. da Recta tangente: $y = mx + b$

Se $x = 1 \Rightarrow y = 1$: $1 = (-1)(1) + b \Leftrightarrow b = 2$

$\boxed{y = -x + 2}$

c) $\left.\begin{array}{l} r_1 : y = m_1 x + b_1 \\ r_2 : y = m_2 x + b_2 \end{array}\right|$ $r_1 \perp r_2$ se $m_2 = -\dfrac{1}{m_1}$

$r_1 : y = -x + 2$ com $m_1 = -1$

$m_2 = -\dfrac{1}{-1} = 1$

Se $x = 1 \Rightarrow y = 1$: $1 = (1)(1) + b_2 \Leftrightarrow b_2 = 0$

$\boxed{y = x}$

2) a) $\int_{-1}^{1} 2x - x^3 + 2x^4 \, dx$

$$\int 2x - x^3 + 2x^4 \, dx = 2\int x \, dx - \int x^3 dx + 2\int x^4 dx$$

$$= 2\frac{x^2}{2} - \frac{x^4}{4} + 2\frac{x^5}{5} + c = x^2 - \frac{x^4}{4} + \frac{2}{5}x^5 + c$$

$$\int_{-1}^{1} 2x - x^3 + 2x^4 \, dx = \left[x^2 - \frac{x^4}{4} + \frac{2}{5}x^5\right]_{-1}^{1} =$$

$$= \left[(1)^2 - \frac{(1)^4}{4} + \frac{2}{5}(1)^5\right] - \left[(-1)^2 - \frac{(-1)^4}{4} + \frac{2}{5}(-1)^5\right]$$

$$= \left(1 - \frac{1}{4} + \frac{2}{5}\right) - \left(1 - \frac{1}{4} - \frac{2}{5}\right) = 1 - \frac{1}{4} + \frac{2}{5} - 1 + \frac{1}{4} + \frac{2}{5}$$

$$= \frac{2}{5} + \frac{2}{5} = \frac{2+2}{5} = \frac{4}{5}$$

b) $\int_{1}^{2} \frac{x+1}{x^2} \, dx$

$$\int \frac{x+1}{x^2} dx = \int \left(\frac{x}{x^2} + \frac{1}{x^2}\right) dx = \int \frac{1}{x} dx + \int \frac{1}{x^2} dx$$

$$= \ln|x| + \int x^{-2} dx = \ln|x| + \frac{x^{-2+1}}{-2+1} + c$$

$$= \ln|x| - \frac{1}{x} + c$$

$$\int_{1}^{2} \frac{x+1}{x^2} dx = \left[\ln|x| - \frac{1}{x}\right]_{1}^{2} = \left(\ln(2) - \frac{1}{2}\right) - \left(\ln(1) - \frac{1}{1}\right)$$

$$= \ln(2) - \frac{1}{2} - 0 + 1 = \ln(2) + \frac{1}{2}$$

c) $\int_0^1 x e^{5x} \, dx$

vamos primitivar por partes $\int x e^{5x} dx =$

$$\int x e^{5x} dx = \frac{1}{5} \int 5 e^{5x} x \, dx$$

$$= \frac{1}{5} \left(e^{5x} x - \int e^{5x} x' \, dx \right)$$

$$= \frac{1}{5} \left(e^{5x} x - \int e^{5x} dx \right)$$

$$= \frac{1}{5} \left(e^{5x} x - \frac{1}{5} \int 5 e^{5x} dx \right)$$

$$= \frac{1}{5} \left(e^{5x} x - \frac{1}{5} e^{5x} \right) + C$$

$$= \frac{x e^{5x}}{5} - \frac{e^{5x}}{25} + C$$

$$\int_0^1 x e^{5x} dx = \left[\frac{x e^{5x}}{5} - \frac{e^{5x}}{25} \right]_0^1 =$$

$$= \left(\frac{(1) e^{5(1)}}{5} - \frac{e^{5(1)}}{25} \right) - \left(\frac{(0) e^{5(0)}}{5} - \frac{e^{5(0)}}{25} \right)$$

$$= \frac{e^5}{5} - \frac{e^5}{25} - 0 + \frac{1}{25}$$

$$= e^5 \left(\frac{1}{5} - \frac{1}{25} \right) + \frac{1}{25}$$

$$= e^5 \left(\frac{5-1}{25} \right) + \frac{1}{25}$$

$$= \frac{4}{25} e^5 + \frac{1}{25}$$

$$= \frac{4e^5 + 1}{25}$$

d) $\int_1^2 x \ln(x^2)\, dx$

Primitivar por partes $\int x \ln(x^2)\, dx =$

$= \frac{x^2}{2}\ln(x^2) - \int \frac{x^2}{2}(\ln(x^2))'\, dx$

$= \frac{x^2}{2}\ln(x^2) - \frac{1}{2}\int x^2\left(\frac{2x}{x^2}\right) dx$

$= \frac{x^2}{2}\ln(x^2) - \frac{1}{2}\int 2x\, dx$

$= \frac{x^2}{2}\ln(x^2) - \frac{x^2}{2} + C$

$\int_1^2 x \ln(x^2)\, dx = \left[\frac{x^2}{2}\ln(x^2) - \frac{x^2}{2}\right]_1^2$

$= \left(\frac{2^2}{2}\ln(2^2) - \frac{2^2}{2}\right) - \left(\frac{1^2}{2}\ln(1^2) - \frac{1^2}{2}\right)$

$= 2\ln(4) - 2 - \frac{1}{2}(0) + \frac{1}{2}$

$= 2\ln(4) - 2 + \frac{1}{2}$

$= 2\ln(4) - \frac{3}{2} = 2\ln(2^2) - \frac{3}{2} = 4\ln(2) - \frac{3}{2}$

e) $\int_{-1}^0 18x^2(x^3+2)^5\, dx = 6\int_{-1}^0 3x^2(x^3+2)^5\, dx =$

substituição: $u = x^3 + 2$

$\frac{du}{dx} = 3x^2 \Leftrightarrow du = 3x^2\, dx$

$u(-1) = (-1)^3 + 2 = 1$

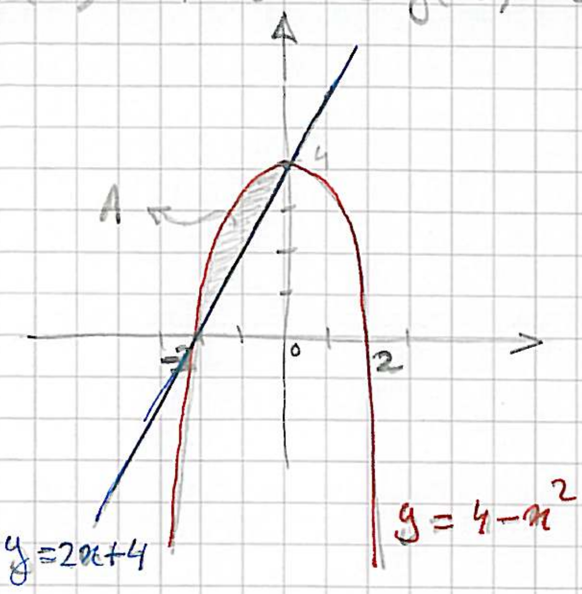
$u(0) = 0^3 + 2 = 2$

$= 6\int_1^2 u^5\, du = 6\left[\frac{u^6}{6}\right]_1^2 = 6\left(\frac{2^6}{6} - \frac{1^6}{6}\right) = 2^6 - 1 =$

$= 2\times2\times2\times2\times2\times2 - 1 = 64 - 1 = \underline{63}$

3) $f(x) = 4 - x^2$ e $g(x) = 2x + 4$

a)

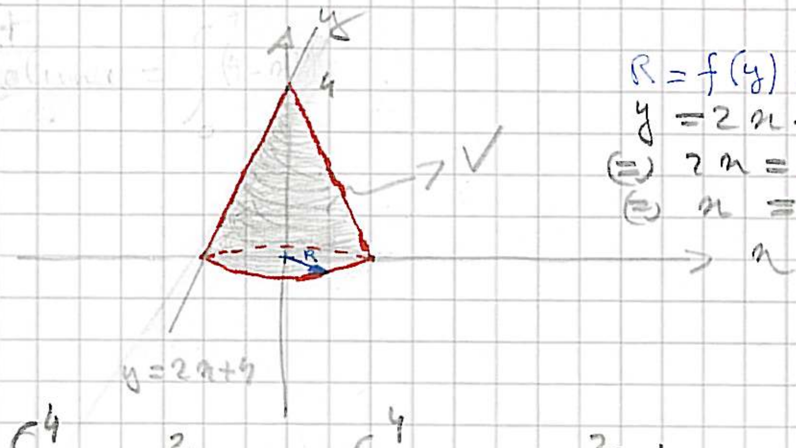


Pontos de interseção:

$$4 - x^2 = 2x + 4$$
$$\Leftrightarrow x^2 + 2x = 0$$
$$\Leftrightarrow x(x+2) = 0$$
$$\Leftrightarrow x = 0 \lor x = -2$$

b)
$$A = \int_{-2}^{0} \left[(4 - x^2) - (2x + 4)\right] dx$$
$$= \int_{-2}^{0} (-x^2 - 2x)\, dx = -\int_{-2}^{0} (x^2 - 2x)\, dx$$
$$= -\left[\frac{x^3}{3} + x^2\right]_{-2}^{0} = -\left[\left(\frac{0^3}{3} + 0^2\right) - \left(\frac{(-2)^3}{3} + (-2)^2\right)\right]$$
$$= -\left[(0) - \left(-\frac{8}{3} + 4\right)\right] = -\frac{8}{3} + 4 = \frac{-8+12}{3} = \frac{4}{3}$$

c)



$$R = f(y) = x$$
$$y = 2x + 4$$
$$\Leftrightarrow 2x = y - 4$$
$$\Leftrightarrow x = \frac{y-4}{2} = \frac{y}{2} - 2$$

$$V = \int_0^4 \pi f(y)^2\, dy = \int_0^4 \pi\left(\frac{y}{2} - 2\right)^2 dy = \pi \int_0^4 \left(\frac{y}{2}\right)^2 - 4\left(\frac{y}{2}\right) + 4\, dy$$
$$= \pi \int_0^4 \left(\frac{y^2}{4} - 2y + 4\right) dy = \pi \left[\frac{y^3}{12} - y^2 + 4y\right]_0^4$$
$$= \pi \left[\left(\frac{4^3}{12} - 4^2 + 4(4)\right) - \left(\frac{0^3}{12} - 0^2 + 4(0)\right)\right]$$
$$= \pi\left(\frac{64}{12} - 16 + 16\right) = \pi\left(\frac{16}{3} - 16 + 16\right) = \pi\left(\frac{16}{3}\right) = \frac{16}{3}\pi$$