Instituto Politécnico de Bragança
Escola Superior de Tecnologia e Gestão

Análise Matemática I – 2003/04

Soluções da Ficha Prática nº 6

Tema: Primitivas – Regras de Primitivação, Substituições, Primitivação por Partes.

1.

a) $\frac{x^4}{4}+c$

b) x^4-x^3+5x+c

c) $\frac{x^7}{7}+\frac{3x^5}{5}+x^3+x+c$

d) $-\frac{1}{x+1}+c$

e) $\frac{(x^2+2)^4}{4}+c$

f) $\frac{(x^3+1)^5}{15}+c$

g) $\frac{3\sqrt[3]{(5x)^2}}{10}+c$

h) $\frac{3}{8}(1+5x^2)^{4/5}+c$

i) $\frac{1}{(x-1)^2}+c$

j) $2\,ln|x|+c$

k) $ln|x+3|+c$

l) $\frac{(\ln(x))^2}{2}+c$

m) $\frac{1}{2}e^{2x}+c$

n) $\frac{2^x}{\ln(2)}+c$

o) $sen(2x)+c$

p) $-3\cos(\frac{x}{3})+c$

q) $\frac{(arctg(x))^2}{2}+c$

r) $4arcsen(x)+c$

2.

a) $2(\frac{x\sqrt{x}}{3}+\sqrt{x})+c$

b) $\frac{(\ln(x))^2}{2}+c$

3.

a) $\frac{(2x+1)\sqrt{2x+1}}{3}+c$

b) $\frac{\ln(3^x+3^{-x})}{\ln(3)}+c$

c) $-arcsen(\frac{1}{x^2+2})+c=arc\,sec(x^2+2)+c$

d) $2arctg(\frac{1-\sqrt{-x^2+3x+1}}{x})+c=arcsen\left(\frac{2x-3}{\sqrt{13}}\right)+c$

4.

a) $xe^x - e^x + c$

b) $\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + c$

c) $x\ln(x) - x + c$

d) $x\ln^2(x) - 2x\ln(x) + 2x + c$

e) $\left(\frac{2}{5}x^2 - \frac{4}{5}x - \frac{16}{135}\right)\sqrt{2-3x} + c$

f) $-\frac{ln^2(x+1)}{2} + (x+1)ln(x+1) - x + c$

g) $-\frac{\ln^2(x)}{x} - \frac{2\ln(x)}{x} - \frac{2}{x} + c$

h) $\frac{xsen(\ln x) - x\cos(\ln x)}{2} + c$

i) $-\cos(x).x + sen(x) + c$

j) $\frac{-sen(x)\cos(x) + x}{2} + c$

k) $(x^2 - 1)sen(x) + 2x\cos(x) + c$

l) $xarctg(x) - \frac{1}{2}\ln(1+x^2) + c$

m) $\frac{e^x sen(x) + e^x\cos(x)}{2} + c$

5. $f(x) = x^2[2ln(x) - 1] + 3$

Tema: Primitivas – Primitivação de funções racionais.

6.

a) $\frac{1}{2}\ln\left|\frac{x-3}{x+3}\right| + c$

b) $3\ln|x| + 2|x-4| + c$

c) $\frac{1}{3}\ln|x-2| - \frac{4}{3}\frac{1}{x-2} + c$

d) $\frac{25}{4}\ln|x| + \left(\frac{25}{6}\sqrt{3} - \frac{45}{8}\right)\ln\left|x - \frac{2\sqrt{3}}{3}\right| - \left(\frac{25}{6}\sqrt{3} + \frac{45}{8}\right)\ln\left|x + \frac{2\sqrt{3}}{3}\right| + c$

e) $x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}arctg(x) + c$

f) $\frac{3}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln\left(x^2+3\right) + c$

g) $\frac{x^2}{2} + x + \ln|x-2| - \frac{1}{2}\ln|x^2+4| - arctg\left(\frac{x}{2}\right) + c$

h) $x^2 + \ln|x-1| + \frac{3}{2}\ln|x^2+1| + c$

i) $\frac{x^2}{4} + \frac{x}{2} + \frac{9}{20}\ln|x-1| - \frac{9}{40}\ln|x^2+9| - \frac{3}{20}arctg\left(\frac{x}{3}\right) + c$

j) $\frac{x^5}{10} - \frac{x^4}{16} + \frac{x^3}{24} - \frac{x^2}{32} + \frac{x}{32} - \frac{1}{64}\ln|2x+1| + c$

k) $\ln\left(x^2+1\right) + \frac{1}{x^2+1} + c$

l) $\ln|t| - \frac{1}{t} - \frac{1}{2}\ln\left(t^2+1\right) - arctg(t) + c$

m) $\frac{\ln|x|}{2} - \frac{\ln\left|x^2-2x+2\right|}{4} + \frac{3}{2}arctg\left(x-1\right) + c$

n) $\frac{x^2}{2} - 2x + \frac{1}{6}\ln|x-1| - \frac{1}{2}\ln|x+1| + \frac{16}{3}\ln|x+2| + c$

o) $\frac{x}{3} - \frac{1}{2}\text{arctg}\left(\frac{x}{2}\right) + c$

7. $arctg\left(3x+1\right) + \pi$

8. $\frac{4}{x+1} + 1$

Tema: Primitivas – Primitivação de funções trigonométricas. Substituições trigonométricas.

9.

a) $\frac{5}{2}sen^2\left(\frac{x}{5}\right) + c$

b) $\frac{sen^3(x)}{3} + c$

c) $-\frac{1}{2}\ln|\cos(2x)| + c$

d) $\frac{1}{2}\ln\left|sen(x^2)\right| + c$

e) $-\frac{1}{3}\cot g^3(x) + c$

f) $3\ln\left|\cos ec\left(\frac{x}{3}\right) - \cot g\left(\frac{x}{3}\right)\right| + c$

g) $\ln|\sec(x)| + x + c$

h) $2\ln\left|\sec\sqrt{x} + tg\sqrt{x}\right| + c$

i) $\frac{1}{2}x - \frac{1}{20}sen(10x) + c$

j) $sen(x) - \frac{1}{3}sen^3(x) + c$

k) $\frac{1}{2}\left(tg(x)\sec(x) + \ln\left|\sec(x) + tg(x)\right|\right) + c$

l) $\frac{1}{3}sen^3(x) - \frac{1}{5}sen^5(x) + c$

m) $\frac{1}{3}\left[\frac{1}{5}sen^5(3x) - \frac{2}{7}sen^7(3x) + \frac{1}{9}sen^9(3x)\right] + c$

n) $\frac{1}{4}\cos^8\left(\frac{x}{2}\right) - \frac{1}{3}\cos^6\left(\frac{x}{2}\right) + c$

o) $\frac{1}{16}x - \frac{1}{128}sen(8x) + \frac{1}{96}sen^3(4x) + c$

p) $\frac{1}{8}x - \frac{1}{32}sen(4x) + c$

q) $\frac{1}{2}\left[\frac{1}{3}\sec^3(2x) - \sec(2x)\right] + c$

r) $\frac{1}{2}sen(x) - \frac{1}{10}sen(5x) + c$

s) $-\frac{1}{16}\cos(8x)+\frac{1}{4}\cos(2x)+c$

t) $\frac{1}{12}sen(6x)+\frac{1}{4}sen(2x)+c$

u) $\frac{2}{3}sen(x)\sqrt{sen(x)}+c$

v) $\frac{1}{4}tg^4(x)-\frac{1}{2}tg^2(x)+\ln\left|\cos(x)\right|+c$

w) $-2\sqrt{2}\cos\left(\frac{x}{2}\right)+c$

x) $\frac{4}{3}\sqrt{2}sen\left(\frac{3x}{2}\right)-\frac{4}{9}\sqrt{2}sen^3\left(\frac{3x}{2}\right)+c$

y) $-\frac{\sqrt{2}}{2}\ln\left|\cos ec\left(\frac{\pi}{4}-x\right)-\cot g\left(\frac{\pi}{4}-x\right)\right|+c$

z) $\frac{1}{18}\sec^6(3x)-\frac{1}{12}\sec^4(3x)+c$

α) $\frac{\sec^2(x)}{2}+c$

β) $-sen(x)+\ln\left(\frac{1+sen(x)}{\cos(x)}\right)$

10.

a) $\frac{\sqrt{x^2-4}}{4x}+c$

b) $-\frac{\sqrt{x^2+9}}{9x}+c$

c) $-\frac{\sqrt{4-x^2}}{4x}+c$

d) $\frac{x\sqrt{x^2+5}}{2}+\frac{5}{2}\ln\left|\frac{x+\sqrt{x^2+5}}{\sqrt{5}}\right|+c$

e) $\frac{x}{9\sqrt{9-x^2}}+c$

f) $-\frac{2}{1+tg\frac{x}{2}}+c$

g) $\frac{x\sqrt{x^2-4}}{2}+2\ln\left|x+\sqrt{x^2-4}\right|+c$

h) $3\ln\left|\frac{3-\sqrt{9-4x^2}}{x}\right|+\sqrt{9-4x^2}+c$

i) $5\ln\left|\frac{5-\sqrt{25-x^2}}{x}\right|+\sqrt{25-x^2}+c$

j) $tg\left(\frac{x}{2}\right)+\log\sec^2\left(\frac{x}{2}\right)+c$

k) $-\frac{\sqrt{9-2x^2}}{9x}+c$

l) $\frac{2-x^2}{\sqrt{1-x^2}}+c$

m) $\frac{1}{2}arcsen\left(\frac{x}{\sqrt{2}}\right)-\frac{x\left(1-x^2\right)\sqrt{2-x^2}}{4}+c$

11.