Consultar apontamentos e exercícios resolvidos, na página da disciplina

1. Resolver o Exercício 63 (d) (f) (g) (i).
2. Resolver o Exercício 73 (d).
3. Resolver o Exercício 75 (f) (o) (p).
4. Resolver o Exercício 79 (a) (b) (c).

Exercício 63. Calcular as transformadas de Laplace das funções usando uma tabela de transformadas (tabela na página da disciplina)

(d) $f(t) = t/2$

$$\mathcal{L}(t/2) = \frac{1}{2}\mathcal{L}(t) \underset{F.11,\, n=1}{=} \frac{1}{2s^2}$$

(f) $f(t) = t^2 e^{-t}$

Usando a fórmula 7 da tabela, com $f(t) = e^{-t}$ (não confundir com a designação da função no enunciado),

$$F(s) = \mathcal{L}(e^{-t}) \underset{F.15,\, a=-1}{=} \frac{1}{s+1},$$

temos

$$\mathcal{L}(t^2 e^{-t}) = (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)$$

$$= \frac{d}{ds}\left(\frac{-1}{(s+1)^2}\right) = \frac{2}{(s+1)^3}$$

<u>Resolução alternativa</u>: usar o teorema da translação em s

$$e^{at} f(t) = F(s-a) \qquad F.\,2,\ a=-1$$
$$f(t) = t^2$$

$$F(s) = \mathcal{L}(t^2) \underset{\substack{F.11 \\ n=2}}{=} \frac{2}{s^3}$$

$$F(s-a) = F(s+1) = \frac{2}{(s+1)^3}$$

(g) $f(t) = e^{-5t}\sin(8t)$

$$\mathcal{L}(e^{-5t}.\sin(8t)) \underset{\substack{F.22 \\ a=-5 \\ k=8}}{=} \frac{8}{(s+5)^2+64}$$

(i) $f(t) = \sin(t+\frac{\pi}{4})$

$$\mathcal{L}(\sin(t+\tfrac{\pi}{4})) = \mathcal{L}\Big(\sin(t)\cdot \underbrace{\cos(\tfrac{\pi}{4})}_{\frac{\sqrt{2}}{2}} + \cos(t)\cdot \underbrace{\sin(\tfrac{\pi}{4})}_{\frac{\sqrt{2}}{2}}\Big)$$

$$= \frac{\sqrt{2}}{2}\left(\mathcal{L}(\sin t) + \mathcal{L}(\cos(t))\right)$$

$$\underset{\substack{F\,13,14 \\ k=1}}{=} \frac{\sqrt{2}}{2}\ \frac{1+s}{s^2+1}$$

Exercício 73. Determinar as transformadas de Laplace

(d) $\mathcal{L}(2\dot{x} - x + 2)$, $x(0) = 1$

$$\mathcal{L}(2\dot{x} - x + 2) = 2\,\mathcal{L}(\dot{x}) - \mathcal{L}(x) + 2\mathcal{L}(1)$$

$$= 2\left(s X(s) - \underset{\overset{\shortparallel}{1}}{x(0)}\right) - X(s) + \frac{2}{s}$$

$$= (2s - 1)\, X(s) + \frac{2}{s} - 2$$

Exercício 75. Determinar as transformadas inversas de Laplace.

(f) $F(s) = \dfrac{1}{s^2}$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) \underset{F.11,\ n=1}{=} t$$

(o) $F(s) = \dfrac{2}{s-1} - \dfrac{3}{s^2+5}$

$$\mathcal{L}^{-1}\left(\frac{2}{s-1} - \frac{3}{s^2+5}\right) = 2\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - 3\,\mathcal{L}^{-1}\left(\frac{1}{s^2+5}\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = e^t \quad , F\ 15,\ a=1$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+5}\right) = \frac{1}{\sqrt{5}}\mathcal{L}^{-1}\left(\frac{\sqrt{5}}{s^2+5}\right)$$

$$= \frac{1}{\sqrt{5}} \sin(\sqrt{5}\, t)$$

F.13
$k=\sqrt{5}$

$\therefore$

$$\mathcal{L}^{-1}\left(\frac{2}{s-1} - \frac{3}{s^2+5}\right) = 2e^t - \frac{3}{\sqrt{5}}\sin(\sqrt{5}\, t)$$

(p) $F(s) = \dfrac{1-e^{-3s}}{s^2}$

$$\mathcal{L}^{-1}\left(\frac{1-e^{-3s}}{s^2}\right) = \underbrace{\mathcal{L}^{-1}\left(\frac{1}{s^2}\right)}_{=\ t} - \underbrace{\mathcal{L}^{-1}\left(\frac{e^{-3s}}{s^2}\right)}_{e^{-3s}\cdot\frac{1}{s^2}}$$

F 11, n=1

$e^{-3s}\cdot\frac{1}{s^2} \xrightarrow{\mathcal{L}^{-1}} u(t-3)(t-3)$

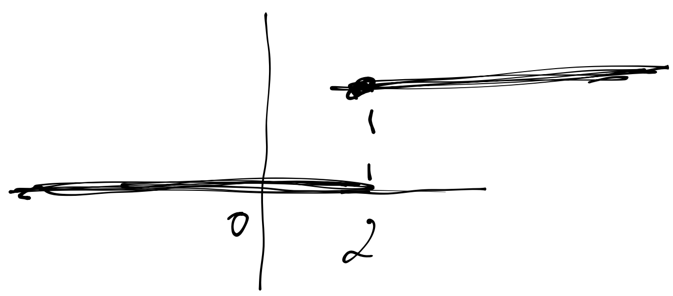
F4, a=3

$$= t - u(t-3)(t-3)$$

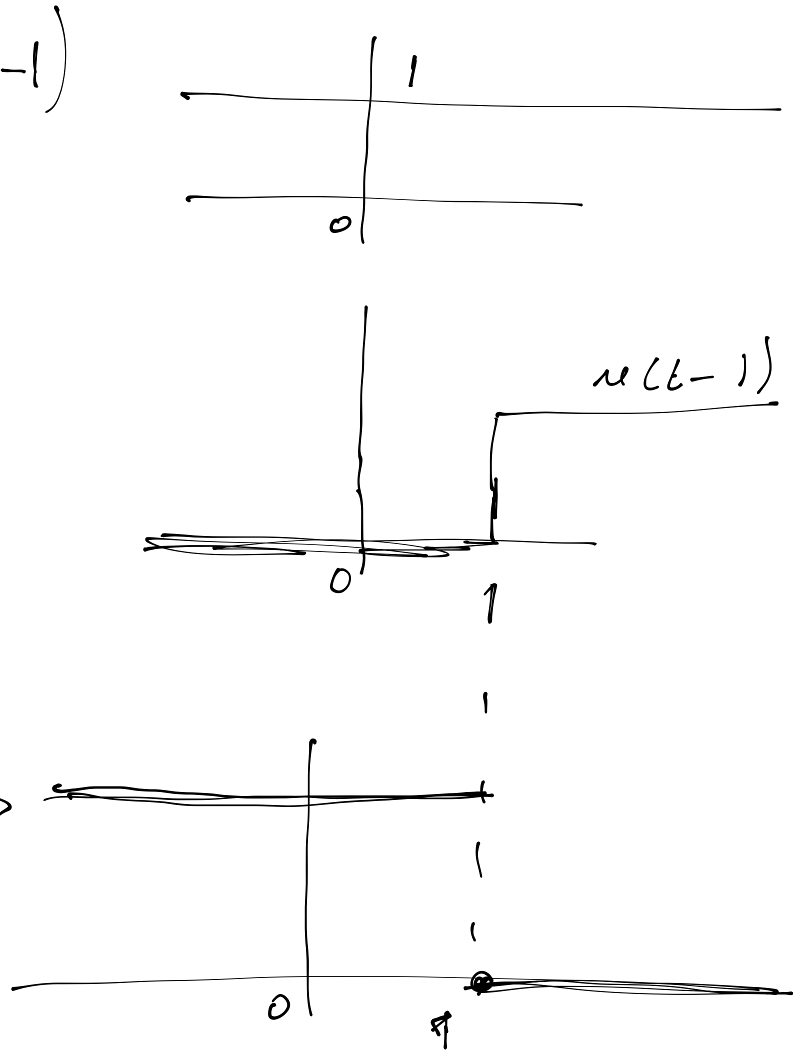
$$= \begin{cases} t, & 0 \le t < 3 \\ 3, & t \ge 3 \end{cases}$$

Exercício 79. Esboçar os gráficos das funções

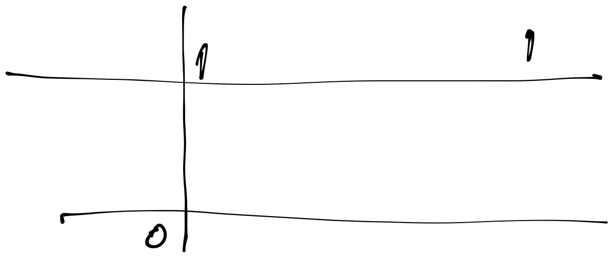
(a) $u(t-2)$

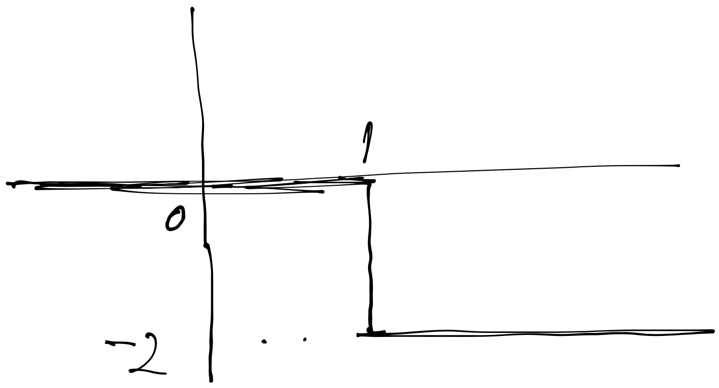


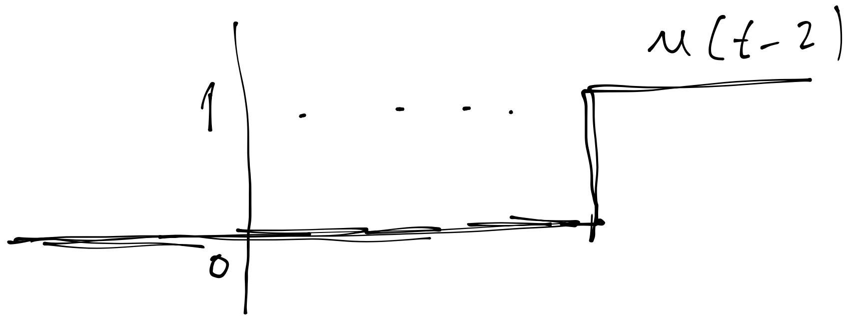
(b) $1-u(t-1)$

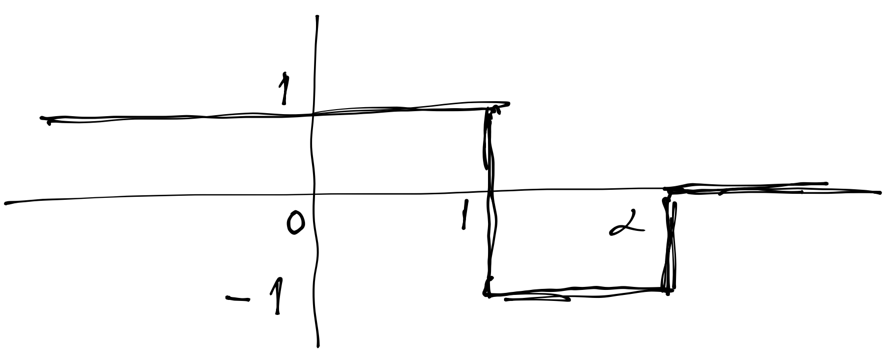


(c) $1-2u(t-1)+u(t-2)$


1
1
0


1
0
-2


u(t-2)
1
0


1
0
1
2
-1